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OBSERVATIONS CONCERNING THE PHYSICAL NATURE OF HALLEY'S COMET, AND RESULTANT COMMENTS

F.W. Bessel

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16 Abstract Observed characteristics of Halley's Comet itself are described as a sequel to reported characteristics of its motion. A wedge of material emanating in the general direction of the sun appears to oscillate about an axis toward the sun. Two hypotheses designed to explain the behavior of material leaving the nucleus of the comet are discussed: the existence of an ether whc density relative to that of comet particles determines their behavior, and the existence of a (polar) force different from the sun's normal gravitational attraction. The behavior of Halley's Comet and several others appears to favor the latter hypothesis.

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OBSERVATIONS CONCERNING THE PHYSICAL NATURE OF HALLEY'S COMET, AND RESULTANT COMMENTS

F.W. Bessel

During the first period in which Halley's Comet was visible, /185* it generated such striking phenomena that it was impossible for me to deny them my attention. The series of observations concerning its apparent movement in the celestrial sphere was therefore accompanied by a second, concerned with its nature. The latter made it necessary to relate the obtained data to one another and to interpret these observations to the point where their possibilities can be assessed. This had not been done earlier, and could not have been done, because information on the phenomena was lacking. At the time of the appearance of the comet of 1811, Olbers provided us not only with a description of the strikingly shaped tail but also with an explanation of the causes which can produce various forms of comet tails. What I now add is based partly on new observations and partly on a theoretical study of the motion of the particles which form the tails of comets. Some of these things I consider to be adequately demonstrated, and other things to be views which require additional testing, through the careful observation of other comets.

In reporting the latter, it is not my intention to assert my views as established truth. Rather, I intend to present an explanation to make known those items to which attention should be paid when future comet phenomena occur. I believe that we would possess more widely usable observations than we actually do concerning the nature of comets if an interpretation of the observations had been available which could have been subject to refutation or confirmation. Concerning myself, I must admit that

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* Numbers in the margin indicate numbered columns in the foreign text.

my observations regarding the nature of Halley's Comet would have been more complete had I had a study much like the present one before me for testing.

Studies on the nature of comets are more in the realm of /186
physicists than of astronomers. They devolve upon the latter, however, because they are primarily in the possession of relatively powerful telescopes. It has since become known that Mr. Arago, who combines the possessions and the characteristics of the astronomer with those of the physicist, has devoted his attention to the comet. The fact that my observations might also be of use to him in his interpretations is a desire which prompts me to report them immediately.

Section 1

From the time at which it was first perceived, the comet always exhibited such pronounced thickening of its nebulosity at one point, which I shall call the "nucleus" in the following, that although it did not have the appearance of a solid body, it could still be distinguished without difficulty from the surrounding nebulosity. This is the way I saw the comet up to the last days of September and also on October 1.

On October 2, the sky, which had earlier been overcast, cleared at 11:00, and the comet now exhibited a considerable change in its appearance. I had the extremely weak 45x ocular in the telescope of the heliometer; the nucleus appeared so brilliant that it had the appearance of a fixed star of sixth magnitude, and I actually believed that it was in front of such a star and that the light from the latter was shining through it. Meanwhile, I made haste to change the level of magnification, which was too weak for the brightness of the telescope's image and was useful almost solely for seeking out an object, to 179x. Viewed

with this magnification, the nucleus no longer looked like a solid, bright point, but rather like a luminous mass which, while highly compressed, still was indefinitely delimited; its shape was of a remarkable nature, as I shall describe in greater detail. I wished to indicate, above, the impression which the comet made /187 on October 2, since we can thereby arrive at an evaluation of the magnitude of its variation. Nevertheless, we do not observe a heavenly object directly, but always through clear or hazy air: we may thus not consider an observed change in its brightness to be intrinsic until we are convinced that the difference in the transparency of the air is insufficient to explain it. In the present case, however, I believe that the brightness of the comet on October 2, as compared with the preceding days, was too great to be completely explained by the perfect clarity of the air which appeared to occur, but it may not go unmentioned that the sky was very hazy on the preceding as well as several subsequent days, on which the comet likewise failed to exhibit the brightness of October 2. Since I cannot state with certainty whether the altered appearance of the comet on this day is partly ascribable to the greater transparency of the air, I must leave it as an undecided question as to whether the great increase in brightness which was observed on October 2 came about between the first and second, i.e. very rapidly, or whether it took place gradually and only became visible on the second. If the former view were adopted, it would then be indisputable that the comet itself experienced a sudden change, had either generated its own light or had become capable, through greater compaction, of reflecting sunlight more strongly. For the change in the position of the comet relative to the earth and the sun between October 1 and 2 was too insignificant to provide an explanation for as great a change in appearance as was perceived.

The above-mentioned remarkable character which the comet exhibited consisted of an emanation of luminous matter from the

nucleus which formed a circular sector of about 90°, was approximately in the direction of the sun, and could be distinguished to a distance of 12-15" from the center of the nebulous background on which it lay. The first of the twelve drawings which I am enclosing with this article shows the appearance of the comet's head; it is cut off by a line which is perpendicular to the direction from the sun to the comet, so the sun must be assumed to be located vertically above the nucleus of the comet. The remaining drawings are to be viewed in the same manner. The dimensions of the nebulosity could be followed out to 2 or 3 min from the center, in spite of the moonlight. An attempt to measure the positional angle of the axis of this emanation yielded 87°50' (12:42 MT); in view of the uncertainty of the phenomenon's appearance, however, this figure can only be considered an imperfect approximation and guaranteed perhaps to only within 5°. /188

It was cloudy on October 3 and just hazy on October 4. The comet appeared much less brilliant than on the 2nd and also did not have the appearance of a fixed star, even with the instrument's lowest magnification; I could not observe anything of the emanation. On the 5th, the comet appeared even more inconspicuous and likewise without emanation, with the sky even less transparent. It cleared up again on October 8. The comet once more shown so vividly that its nucleus appeared solid even at 120x magnification and lost this appearance only at even higher magnifications. The emanation had become more pronounced than on the 2nd, the angle of its margins smaller, about 45°; I could distinguish it up to a distance of 15-20" from the center of the bright background on which it lay. I determined its positional angle to be 135°20' (11:53 MT), but with hardly any greater certainty than on the 2nd. Its margin on the right side was noticeably curved. The nebulosity surrounding the nucleus was also brighter on this side than on the other. All of these conditions are illustrated in the drawing.

The next clear night was that of October 12. The comet was close to the Earth and when viewed with the naked eye appeared brighter than the stars of second magnitude in Ursa Major. The emanation had become larger and brighter, as the four figures which represent the appearance of the comet on this night show. I could follow it more than 30" from the center. Its curvature toward the right side was more conspicuous than on the 8th. The nucleus of the comet and its emanation gave the appearance of a burning rocket whose tail is deflected to the side by a breeze. If we forget about the curvature on the right side, the two margins of the emanation probably formed an angle of 30°. Since the night remained clear, I was able to follow the comet for 9 hours, from sundown until 3:00 a.m. Its appearance remained almost the same during this time and experienced no important change aside from a reduction in the difference between the brightnesses of the nucleus and the part of the emanation closest to it; this was initially considerable enough that the two could be adequately distinguished; at the time of the last observation, however, it had decreased so that I had some difficulty distinguishing between the center and the beginning of the emanation while making a position determination at this time. The direction in which the emanation appeared thus experienced relatively large changes during this night, as the following measurements of its positional angle show: /189

6h 5' MT	208° 6'
10 34	222 20
12 40	233 58
14 24	250 23

Since the emanation appeared more definite now than on previous nights, these measurements should be more certain than the earlier ones; I actually consider them to be more reliable although the curvature of the emanation on the right side made it so difficult

to judge the position of its axis that I must still assign an uncertainty of several degrees to the positional angles. I would have attempted to measure what one would have been capable of considering to be a solid nucleus of the comet under weaker magnifications, if its margins had not become completely indeterminate when greater magnifications were used: at 179x, I would perhaps have been able to measure a diameter of 2 or 3" if I had wished to measure what still appeared to be bright; 290x magnification destroyed all appearance of a solid body, however, and showed that a size figure would have been completely arbitrary.

On October 13, the first hours of the evening were clear, and the comet exhibited the brightness of the previous night, but a quite different character. An emanation delimited on both sides was no longer present; in its place was an undelimited mass of luminous matter to the left of center, as shown in the drawing. I could estimate its position only quite roughly, since its uncertainty made all measurement impossible: its brightest part preceded the center of the comet to the north (7:00 MT), so its positional angle was greater than 270°; I believe that it was less than 290°, and am therefore of the opinion that an assumed value of 280° for it would not be far from actuality. The luminous matter appeared to have accumulated on the sun side in greater quantity than the night before; there was again more on the right side than on the left. Since the sky was beautifully clear and the moon was absent, there is not the least doubt that the observed great change in the comet had occurred.

October 14 allowed only a clear 1/4 hour, but a beautifully clear one; it was sufficient to indicate major new changes in the comet and allowed the drawing to be drafted. The emanation had not only reappeared, but had become far more vivid and stronger /190
than on the 12th; I could distinguish it even at 45" from the center. On the other hand, the intensity of the nucleus had decreased, so that it had already lost the appearance of solidity at

90x magnification. The curvature of the borders of the emanation were visible on both sides this night; however, the emanating cone was brighter on the right than on the left. The positional angle of its axis could be observed with rather high precision, at 222°25' (7:14 MT).

On October 15, the comet was much less bright than on the preceding night; the emanation was poorly delimited, as shown in the drawing. I found its positional angle to be 176°55' (6:45' MT); with the object poorly delimited, however, I consider this determination to be merely an approximation, uncertain within several degrees. The direction of the comet's tail was pointed (6:56' MT) almost at γ Draconis, passing the star at about 1/2° to the left. I could not note anything certain about a curvature of the tail, which I could follow with certainty up to half the star's distance from the comet, but occasionally believed to see reaching almost to the star. The estimate given of its direction refers to the point on its axis which lay at the center between the comet and the star.

On October 20, only a weak trace of the emanation could be seen, the direction of which I was unable to determine, since it was too indistinct. The luminous matter appeared to be distributed more extensively and uniformly than before. As little as these circumstances are worth explaining by means of a special drawing, I am still including it to elucidate the behavior of the comet as compared with its earlier and later behavior. Otherwise, it serves only to represent the appearance of the emanation; its direction is drawn arbitrarily, since I could no more make an estimate than an observation of it.

On October 22, the comet was very bright and, when seen with the naked eye, was at least comparable to stars of third magnitude. The emanation was more vivid than ever, but had again changed entirely in its form and had the appearance shown in the

drawing; it had assumed a considerable curvature toward both sides and its brightness was much more uniform than before. Its extreme margin on both sides extended beyond a circular arc of about 30" radius (with which it coincided at its apex) and was thus curved in somewhat parabolic fashion; the distance of each of its ends from the center of the comet was 35". I found the positional angle of its center line to be 270°20' (6:03) -- apparently with very little certainty, in that what I could take to be the center line necessarily had to be very indeterminate, as shown in the drawing. The direction of the tail (7:28 MT) was toward σ Ophiuchi; I could follow its length only half the distance from the comet to this star. /191

On October 25, as the weather again became beautifully clear, the nucleus of the comet was so bright that it would have been possible to take it for a fixed star with the weakest magnification of the heliometer as dusk made the nebulosity still almost invisible. The beautiful picture presented by the emanation which had become visible on the 22nd had disappeared and, in its place, one saw only relatively weak accumulations of light on both sides of the center, which I have shown in the drawing. The positional angle of a line perpendicular to these accumulations was 252° (6:07 MT), but this can only be considered an approximation.

On October 28, I saw the comet only in the haze of the horizon, through which I could observe nothing special. On the 29th, its appearance was almost like that of the 25th, but even less distinct, as the low position of the comet would lead us to expect. Later, I observed it again only on November 8, but it was not possible to observe anything notable.

In the descriptions given, I have not always repeated the fact that the right side of the nebulosity surrounding the center was constantly brighter than the left. I have also not included

several other features which would have constantly had to be repeated; less luminous matter appeared to be present to the side of the center facing away from the sun than on that facing it; a darker spot also intruded into the curvature of the nebulosity on the sun side, so that its periphery did not appear uniformly convex. I have sought to show both in the drawings; but I must note that the comparatively great brightness of the nucleus and its emanation have probably caused the background on which the two were observed to appear darke than it really was; the above-mentioned interruption in the curvature of the nebulosity's margin at the apex was not very conspicuous. Finally, I would like to mention that several directions from the nucleus occasionally appeared to me to contain more nebulosity than others; I have devoted little attention to this, however, since I consider /192 them to be slight modifications of brightness of which, moreover, I could not be quite certain.

Section 2

The most remarkable thing which the comet exhibited is without a doubt the rotating or oscillating motion of the emanating cone of light which occurs both between the consecutive observations during the night of October 12 and between the isolated observations on the other dates. Nothing similar had previously been perceived; this demonstrates not so much that this was not visible for other comets, as that it was not observed.

In order to subject this phenomenon to an overview, I shall first compile the observed positional angles of the axis of the emanation and for each of these describe the positional angle of the great circle drawn from the comet to the sun, as well as the difference between the two, or the angle between the directions of the sun and the emanation.

	MT	Positional angle of Emanation	Sun	Difference
Octbr. 2	12h42′	87°50′	95°41′6	— 7°52′
8	11 53	135 20	116 10,8	+19 9
12	6 5	208 6	189 13,6	+18 52
	10 34	222 20	192 53,3	+29 27
	12 40	233 58	194 32,0	+39 26
	14 24	250 23	195 51,9	+54 31
13	7 0	280 0	207 9,3	+72 51
14	7 14	222 25	219 51,3	+ 2 34
15	6 45	176 55	229 2,5	—52 8
22	6 3	270 20	253 36,0	+16 44
25	6 7	252 0	256 17,6	— 4 18

[Note: Commas in numerals are equivalent to decimal points.]

It can be seen from this that the emanating cone of ight has moved considerably from the direction toward the sun, both to the right and to the left, but has always returned to this direction, only to go over to the other side. It was a happy circumstance that the comet could be observed continuously on October 12 and the nature of its motion clearly recognized: as this night began, the emanation was already noticeably to the left of the direction toward the sun, and as it progressed, the emanation moved considerably farther to the left; on the 13th, it was found to have moved even farther to this side; on the 14th, it had returned almost to the direction of the sun, and on the 15th it had moved a significant amount toward the right. The problem was now to determine, from the observations, the axis about which the emanating cone of light rotated.

I shall therefore seek the relationship between the elements of the rotation of the emanating cone of light's axis and the positional angle at which this axis is found. If we construct straight lines from the center of the comet in the directions of the axis of rotation, the axis of emanation and the earth and label the points in the celestrial sphere corresponding to these directions A, B, C, then the side AB of spherical triangle ABC

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is the angle between the axis of rotation and the axis of the emanation (θ); side AC is the distance (S) of the earth from the pole of rotation; side BC is the distance ( ) of the earth from the axis of the emanation. Moreover, angle A is that between two planes laid through the axis of rotation, one through the axis of the emanation, the other through the earth; or, if the angle between the first plane and the plane of the circle of declination of the axis of rotation, as measured from the north pole of the equator, is designated u and the positional angle of the position, as viewed from the comet, of the earth at the pole of rotation is designated P', then A = u - P'. Finally, C is the angle between two planes laid through the comet and the earth, one of which passes through the axis of rotation and the other through the axis of the emanation; or, if the positional angle of the pole of rotation at the geocentric location of the comet is designated P and the positional angle at which the axis of the emanation appears is designated p, then C = P - p. Thus from the formulas of spherical trigonometry we have

$$\begin{aligned} \cos T &= \cos S \cos\theta + \sin S \sin\theta \cos(u - P') \\ \sin T \cos(P - p) &= \sin S \cos\theta - \cos S \sin\theta \cos(u - P') \\ \sin T \sin(P - p) &= \sin\theta \sin(u - P') \end{aligned}$$

If we assume a point on the axis of the emanation at distance r from the center of the comet and designate the distances of the comet and this point from the earth ρ and ρ' and designate the angle at which r appears as s, we then have

$$\begin{aligned} \rho' \cos s &= \rho - r \cos T \\ \rho' \sin s &= r \sin T, \end{aligned}$$

and thus

$$\begin{aligned} \rho' \cos s &= \rho - r(\cos S \cos\theta + \sin S \sin\theta \cos(u - P')) \\ \rho' \sin s \cos(P - p) &= r(\sin S \cos\theta - \cos S \sin\theta \cos(u - P')) \\ \rho' \sin s \sin(P - p) &= r \sin\theta \sin(u - P'). \end{aligned}$$

The relationship being sought is indicated by these formulas. If we designate the comet's geocentric right ascension and deviation α and δ and the right ascension and deviation of the pole of rotation A and D, we then obtain the following formulas for determining S, P and P':

$$\begin{aligned}
\cos S &= -\sin\delta \sin D - \cos\delta \cos D \cos(A-\alpha)\\
\sin S \cos P &= \cos\delta \sin D + \sin\delta \cos D \cos(A-\alpha)\\
\sin S \sin P &= \cos D \sin(A-\alpha)\\
\sin S \cos P' &= -\sin\delta \cos D + \cos\delta \sin D \cos(A-\alpha)\\
\sin S \sin P' &= -\cos\delta \sin(A-\alpha)
\end{aligned}$$

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or the Gaussian trigonometric formulas:

$$\begin{aligned}
\sin\tfrac{1}{2}S \sin\tfrac{1}{2}(P+P') &= \sin\tfrac{1}{2}(A-\alpha)\sin\tfrac{1}{2}(D+\delta)\\
\sin\tfrac{1}{2}S \cos\tfrac{1}{2}(P+P') &= -\cos\tfrac{1}{2}(A-\alpha)\cos\tfrac{1}{2}(D-\delta)\\
\cos\tfrac{1}{2}S \sin\tfrac{1}{2}(P-P') &= -\sin\tfrac{1}{2}(A-\alpha)\cos\tfrac{1}{2}(D+\delta)\\
\cos\tfrac{1}{2}S \cos\tfrac{1}{2}(P-P') &= -\cos\tfrac{1}{2}(A-\alpha)\sin\tfrac{1}{2}(D-\delta)
\end{aligned}$$

The formulas are suitable for comparing arbitrary assumptions concerning the direction of the axis of rotation with observations of the positional angle of the emanation's axis. Since r is very small here compared to ρ, we can set ρ' = ρ; if we designate the foreshortening of r as n or set

$$\sin\varepsilon = n\cdot\frac{r}{\rho}$$

we have the following for obtaining this foreshortening and the positional angle:

$$\begin{aligned}
n\cos(p-P) &= \sin S\cos\theta - \cos S\sin\theta\cos(u-P')\\
n\sin(p-P) &= -\sin\theta\sin(u-P')
\end{aligned}$$

Section 3

I shall first compare the observations with the assumption that the emanation rotates in the plane of the comet's orbit or

experiences oscillations about an axis which is perpendicular to this plane. If we designate the location of the ascending node of the orbit on the equator N and its inclination relative to the equator I, then

$$A = N - 90^\circ, \quad D = 90^\circ - I$$

is assumed.

According to the elements derived by Professor Rosenberger from the appearance of the comet in 1759 and reduced by him to the present time by perturbation calculations, which I (Astronomische Nachrichten, No. 293) showed to correspond very closely to the present phenomenon if we assume the time of passage through perihelion to be November 15.94984 (Paris meridian), node length and inclination, referred to the ecliptic and the true equinox of October 12, are

$$55^\circ 11' 3.7'' \text{ and } 162^\circ 16' 35.4'',$$

which assumption, combined with the true obliquity of the ecliptic = 23°27'43"3, yields

$$N = 131^\circ 19' 50.8'', \quad I = 160^\circ 33' 32.4''$$

i e. /195

$$A = 41^\circ 19' 50.8'', \quad D = -70^\circ 33' 32.4''.$$

I have calculated the geocentric locations of the comet for the times of my observations, also required for the comparison of observations, from the same elements and report them here along with the locations of the sun.

MT	Paris.	Lg dist Comet ☉	Comet's AR.	Comet's Decl.	Sun's AR.	Sun's Decl.
			° ′ ″	° ′ ″	° ′ ″	° ′ ″
Oct.	2,47871	9,64004	102 50 51	+41 17 1	188 15 0	— 3 35 6
	8,44469	9,39190	130 34 59	58 24 30	193 44 17	— 3 53 6
	12,29302	9,27100	205 38 57	57 44 48	197 11 44	— 7 18 43
	12,38982	9,27043	209 5 2	56 24 58	197 22 6	— 7 22 56
	12,47732	9,27039	210 36 31	55 45 39	197 26 57	— 7 24 54
	12,54955	9,27038	211 49 47	55 12 25	197 30 57	— 7 26 32
	13,24126	9,27646	221 51 14	49 26 8	198 9 21	— 7 42 8
	14,25094	9,30033	232 5 35	40 29 7	199 5 32	— 8 4 48
	15,23080	9,33604	238 43 42	32 21 54	209 0 11	— 8 26 41
	22,20163	9,63440	255 8 25	2 19 40	206 33 9	—10 58 47
	25,20441	9,73284	257 12 22	— 2 45 6	209 24 51	—12 1 59

From this we obtain the quantities appearing in the formulas:

	S	P	P'
Octbr. 2	59°48'7	199° 17 0	130°10'1
8	36 46,7	213 46,2	128 58,4
12	14 25,3	201 10,5	35 23,5
	15 5,6	195 43,9	26 46,9
	15 30,8	193 23,2	23 2,3
	15 54,4	191 33,8	20 6,2
13	21 7,5	179 31,0	359 3,4
14	30 34,8	172 58,9	343 47,3
15	39 21,3	171 2,6	336 36,5
22	71 39,9	168 44,0	324 8,8
25	77 2,9	168 27,3	323 6,2

Since $\theta = 90°$ under the assumption of oscillation of the emanation in the plane of the comet's orbit, which is being pursued here, the formulas at the end of the preceding section become

$$n\cos(p-P) = -\cos(u-P')\cos\delta$$
$$n\sin(p-P) = -\sin(u-P')$$

In these I have used both the value observed and the value indicating the direction of the sun for p, i.e. have obtained two values for u, one of which is referred to the axis of the emanation, the other, $u^{(o)}$, to the direction toward the sun. The

difference between the two is the angle which is enclosed by the axis of the emanation and the radius vector.

	u	u°	$u-u^\circ$	n
Octbr. 2	161°27'6	193°38'6	−12°11'	0,841
8	223 17,8	199 31,3	+23 47	0,989
12	222 6,0	203 48,6	+18 17	0,969
	232 35,2	204 2,2	+28 33	0,972
	242 30,4	204 8,0	+38 22	0,977
	257 55,4	204 14,4	+51 41	0,989
	u	u°	$u-u^\circ$	n
13	240° 16'5	205° 5'3	+35°11	0,998
14	208 57,0	206 22,5	+ 2 35	0,933
15	162 19,6	207 39,8	−45 20	0,773
22	267 14,3	218 9,9	+49 3	0,855
25	206 18,1	223 27,2	−17 9	0,938

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Some attention to the sequence of values $u - u^\circ$ (the angle between the emanation and the radius vector of the comet, taken to be positive if the former, in terms of the direction of motion, is ahead of the comet) shows that they can be explained to some extent by an oscillating motion of the emanation whose period is 4.6 days and whose extent is 60°. If we count time t from October 14.2825 on, the formula which provides this interpretation is

$$u = u^\circ - 60^\circ \sin\left\{t \cdot \frac{360^\circ}{4{,}6}\right\}$$

How this represents the observations of positional angles can be seen from the following comparison of the angles calculated from it and those observed between the emanation and the direction of the sun.

	Calc.	Obs.	Diff.
Octbr. 2	−17°31′	− 7°52′	− 9°39′
8	+41 36	+19 9	+22 27
12	+18 19	+18 52	− 0 33
	+32 36	+29 27	+ 3 9
	+38 29	+39 26	− 0 57
	+42 52	+54 31	−11 39
13	+58 3	+72 51	−14 48
14	+ 2 34	+ 2 34	0 0
15	−66 36	−52 8	−14 28
22	+21 31	+16 44	+ 4 47
25	− 7 41	− 4 18	− 3 23

A portion of these differences can immediately be ascribed to the unavoidable uncertainty in observations of a phenomenon which was as ill-defined as the emanation generally was. I believe that we could do this on October 15, when uncertainty was extremely great. The observation made on October 13 actually should not have been put in the series of those to be compared, since the object to be observed, namely the emanation itself, was not present, and another object, an accumulation of luminous matter, was taken in its place. The last observation on the 12th will in any case provide little evidence against the correctness of a formula with which three other observations on the same day agree, to the extent called for by their own uncertaintly. I consider only the observation of October 8 to deviate decisively from the formula and would view it as evidence of its incorrectness if I considered it to be justifiable to demand complete constancy on the part of the period and amplitude of the oscillations. /197 Aside from this, we can offer nothing more to theoretically justify the formula than the fact that it provides an approximation for every pendular motion and is the simplest of the periodic formulas.

Section 4

Although the observed movements of the emanation can be explained in this manner, i.e. by assuming their cause to be an

oscillating movement in the plane of the comet's orbit, as well as one could reasonably expect, it still appears necessary to me to also try to learn what the second type of rotary motion about the direction from the comet to the sun could yield. I have thus made a second comparison, between the observations and the assumption that the axis of the emanation intersects the radius vector at a constant angle and rotates uniformly about the latter. It is apparent that the two assumptions would yield exactly the same thing if the earth were located in the plane passing through the sun and perpendicular to the plane of the comet's orbit and if this changed just as little during the time of observations as the angle between the radius vector and the straight line passing from the comet to the earth. They do not yield the same thing, however, if this is not the case. For our comet, the difference must be considerable, since its movement relative to the earth was very great during the time period of observations.

Under the assumption which is now to be pursued, the location of the sun as viewed from the comet is in each case the pole of rotation. I will give it first:

MT Paris.	Log. Dist. Comet ☉	A	D
Oct. 2,47871	0,03498	206° 53′ 37″	−18° 52′ 16″
8,44469	9,99656	200 43 36	−18 16 57
12,20302	9,97088	196 15 12	−17 43 44
12,30982	9,96958	196 1 5	−17 41 47
12,47732	9,96896	195 54 25	−17 40 52
12,54955	9,96846	195 48 54	−17 40 ?
13,24126	9,96359	194 55 43	−17 32 37
14,25094	9,95643	193 35 40	−17 20 56
15,25080	9,94942	192 15 44	−17 8 42
22,20163	9,89802	181 29 8	−15 10 13
25,20441	9,87555	176 7 33	−13 57 55

From the above and from the geocentric positions given for the comet in Section 3, we obtain

	S	P	P'
Octbr. 2	87°17'8	95°41'6	232°12'4
8	84 21,6	116 10,8	209 40,8
12	104 7,2	189 13,6	174 50,7
	105 4,6	192 53,3	172 33,5
	105 30,7	194 32,0	171 28,8
	105 52,0	195 51,9	170 34,8
13	108 53,7	207 9,3	161 51,9
14	111 59,8	219 51,3	149 17,6
15	113 27,7	229 2,5	138 7,3
22	105 5,5	253 36,0	96 32,5
25	99 19,1	256 17,6	69 27,9

Pursuing this assumption has shown that it is not suitable for representing all observations even approximately: each assumed value for the angle between the emanation and the radius vector corresponding in some degree to the observations of October 12, 14 and 15 differs considerably from the earlier and later values. If, for example, we assume $\theta = 55°$ and $u = 70°t$, where t indicates time measured from October 12.1557, we obtain the following comparison between the calculated and observed values for the angle between the emanation and the sun's direction:

	Calc.	Obs.	Diff.	n
Octbr. 2	− 9°13'	− 7°52'	− 1°21'	0,852
8	+52 16	+19 9	+33 7	0,975
12	+18 35	+18 52	− 0 17	0,378
	+42 41	+29 27	+13 14	0,488
	+49 10	+39 26	+ 9 44	0,558
	+52 54	+54 31	− 1 37	0,618
13	+57 19	+72 51	−15 32	0,971
14	+ 2 34	+ 2 34	0 0	0,839
15	−53 8	−52 8	− 1 0	0,998
22	+58 2	+16 44	+41 18	0,987
25	−56 6	− 4 18	−51 48	0,958

This assumption deviates quite considerably from observation on October 8, 22 and 25; other assumed values for θ and u yield no better success. A comparison of the two hypotheses thus favors the first. Yet there are still other reasons which favor oscillation of the emanation in the plane of the orbit.

One of these is based on the length which the emanating cone of light was seen to have on October 12 and 14. On these

dates, it was relatively close to the plane passing through the earth, the comet and the sun; about one-half a period of u lay between the two, or a change in this argument of 180°. In the first hypothesis, this change has no effect on foreshortening; in the other, it produces considerable differences in foreshortening which I have actually found to equal 0.378 and 0.839. If we also consider the change in the distance between the coment and the earth and take the length which the emanation would be seen to have at the time of the first observation on October 12 if it had been positioned perpendicular to the line of sight equal to s, we obtain its apparent length, assuming that it actually remains constant: /199

Hypothesis	I.	II.
Octbr. 12.	0,969.s	0,378.s
14.	0,872.s	0,784.s

On October 12, I was able to follow its length more than 30" from the center; on the 14th, to 45"; the moon appeared very bright on the former date; on the latter it was absent and the sky was quite clear. Without this difference in external circumstances, I would have had to find a smaller difference in the observed lengths. The true size of the emanation itself, on the other hand, was probably greater on the 14th than on the 12th, since the observations show it initially to be smaller and then continually larger until the 22nd, at which time it was almost one and one-half times as large as on the 14th, in spite of the light attenuation which it must have suffered by the comet's being more than twice as far from the earth. If we take all of this into consideration, this appears to be in complete agreement with the first hypothesis, whereas it deviates considerably from the second.

The other reason is based on the appearance of the emanation. The observations permit no doubt that the emanation was more

vivid when it appeared in the direction of the sun than when it deviated considerably from the latter; at one of the maxima in its deviation from this direction, on the 13th, it completely ceased being visible; in the vicinity of the other, on the 15th, it had become extremely inconspicuous, whereas it had a quite vivid appearance on the 12th and 14th. If its motion is to be attributed to oscillation in the plane of the orbit, it is not only conceivable but even probable that it exhibited its greatest vitality when oriented in the direction of its cause, the sun. If, on the other hand, it is the result of rotation about the radius vector, it always maintains the same inclination relative to the sun, and we see no reason which could explain its varying vitality; on the contrary, it would have to have appeared most vividly at the limits of its motion, since it was observed unforeshortened here. Finally, we might also note that rotation about an axis which is not fixed but is always oriented toward the sun is only permissible if a special physical cause is assumed for this movement of the axis.

Since the observed values for the positional angles of the emanation support the correctness of their explanation in terms of an oscillating motion and since unobjectionable reasons of another nature which I have just demonstrated are in agreement with the above, I see no objection to assuming this oscillating motion in the plane of the orbit, as a result of observation. /200

Section 5

Oscillatory movement of the emanation about the direction of the radius vector requires that the sun exert -- in addition to the attractive force which moves the center of gravity of the comet in accordance with Kepler's laws -- an additional rotating force on it whose resultant either passes through a point located on the sun side of the comet's center of gravity, if it is directed toward the sun, or, if it is directed away from the

sun, passes through a point lying on the far side of the center of gravity; or if it has two resultants rather than one, that one of them satisfies the first case and the other the second case. If this were reversed, any deviation occurring on the part of the emanation's axis from the direction toward the sun would have increased and ultimately been transformed into that in the opposite direction.

Moreover, the oscillating motion of the emanation requires that the body of the comet itself possess this motion. Since the particles of emanating matter are not in a fixed relationship, each of these assumes the motion which results from the forces acting upon it and its original state of motion. A combining of these motions of all particles to form an oscillating motion is ruled out if rigidity is assumed on the part of the emanation. It could only be explained by the forces if we were willing to attribute a similar oscillating motion to these themselves, i.e. assume something of which this is a result. Thus it is the original state of motion whose effects we have observed, and it follows from this that the body of the coment has ejected the luminous matter from a portion of its surface and has determined its direction in each case by the direction of this portion.

The sun's usual force of attraction on heavy bodies is not sufficient, however, to explain an oscillation on the part of the comet's body of such a short period as that observed, and it is necessary to seek another cause. It is indeed certain that the parts of the comet closer to the sun are attracted more strongly by it than the more distant portions; and that as a result of this, in conjunction with its motion in a curvilinear orbit, an /201
oscillating motion similar to the true libration of the moon can occur if it presents a lengthened diameter to the sun. But although integration of the known differential equation for libration presents as yet unmastered difficulties in the case of

such an eccentric motion as that of the comet and although -- due to the lack of knowledge of its moments of inertia -- it is even less possible to arrive at a numerical result for the period of oscillation, we can still easily demonstrate that the rapidity of the change in the argument of this motion is a quantity on the order of the square root of the product of the solar mass and the comet's diameter divided by the cube of the comet's distance from the sun. This quantity is thus extremely small, or the period of the motion resulting from the sun's attractive force is extremely long. The observed short-period motion can thus not be explained in this manner.

I see neither how we will be able to extract information from the assumption of a polar force as to which of the comet's radii endeavors to face the sun, the opposite of which endeavors to face away from it, nor what reason could exist for rejecting the assumption of such a force a priori. We do not even lack an analogy, since the earth itself possesses a polarity -- magnetic -- although we still do not know that its opposites relate to the sun. Should this be the case, one effect could be manifested in the precession of the equinoxes.

It cannot be doubted that this force, which appears to be necessary for explaining the short-period oscillations, can provide such explanation in toto. It is a function of the strength of the force; length is a function of an arbitrary constant referring to the initial state.

I would also like to add that if the sun acts upon a portion of the comet's mass with a force other than the usual attraction, i.e. this portion has a greater or lesser attraction or repels it, this special action must necessarily be a polar one, i.e. one which causes the opposite effect on another portion of the mass. If this were not the case, the sum of all forces which the

sun exerts upon the entire mass would not be proportional to this mass, and as a result the motion of the comet in accordance with Kepler's laws would not correspond to the same solar mass that we /202 know from the motions of the planets. This is quite contrary to the observed data, which would have disclosed even a small difference. Thus if we can obtain convincing evidence that the entire mass of the comet is not attracted in the ordinary manner by the sun, we shall then have new evidence of the action of a polar force in the former.

Section 6

Nevertheless, I must provide a more complete presentation of the observations before I attempt to further interpret the phenomena which comets offer. We possess excellent pictures of the comet of 1744 in a special work by Heinsius containing his description. These pictures are based on observations made with a four-leg telescope by Short, an instrument which, if evaluated on the basis of its performance, deserves no less distinction than the other instruments by the same master used so frequently at that time. These are the only available observations of a large and magnificent comet whose nucleus would have offered remarkable peculiarities; the nucleus of the comet of 1811 was quite uninformative compared to that of 1744. The latter was so similar to Halley's Comet in every respect, however, that the one can be a supplement to what the other has shown. In the case of the comet of 1744, what Halley's exhibited on a relatively small scale appeared on a larger one; thus there will be more frequent cause to supplement my observations with those of Heinsius than vice versa. It is to be regretted that Heinsius was not aware of the oscillating motion of the emanation or lacked the means of making proper observations of this.

I could not make any observations regarding the shape of the nucleus: prior to October 2, I could not because the comet still

appeared too distant and too dark; after this date, because the emanation was too bright close to the nucleus to permit an assessment of the shape which it would have exhibited if separated from the emanation; in general, I could not because it was always too small to clearly exhibit its indistinctly outlined shape The comet of 1744, however, had a considerably lengthened diameter presented to the sun, whose ratio to the shortest diameter perpendicular to it is given by Heinsius as 3:2. This lengthened nucleus was apparent before the emanation became visible, on January 5. It maintained its shape even after the emanation had formed, until the end of observation on February 16. The emanation arose at the tip of the longest diameter. I offer this observation in order to make it more probable, through the use of another example, than already indicated by the behavior of Halley's Comet that it is the sun which produces the emanation and turns the portion of the comet's surface from which it originates toward itself. /203

Heinsius first noted the emanation in the direction of the sun on January 25, on which date it appears to have developed to the same extent as that of Halley's Comet on October 8 or 12. On January 31, it was almost the same as that which I observed on October 22, the one difference being that its endeavor to curve away from the sun had already been more successful, and one of its margins had actually curved more in this direction than my observation shows. On February 2, this upward movement of the emanating matter had already become much more apparent and occurred on both margins, so that it formed the beginning of two sides of a tail which continually became more vivid and longer on subsequent dates. The observations made on the two comets thus do not permit the least doubt that the matter which was emanated toward the sun did not continue to move toward the latter but, on the sun side of the comet, reached a maximum in their motion away from it and then proceeded in the opposite direction.

The portion of the comet's surface from which the emanation originated appears in both cases to have become larger as the sun was approached. In the case of Halley's Comet, this is suggested by the observation on October 22 and the later one on the 25th; in the case of the comet of 1744, this is very apparent from Heinsius' drawings. Initially, only a small portion of the surface produced visible emanation; later, this portion became larger and extended farther and farther over the half facing the sun, so that it took up this entire half at the time of last observation on February 16. During the entire time of Heinsius' and my observations, the luminous matter, i.e. that emanating visibly, flowed outward at acute angles to the radius vector, i.e. toward the sun -- this is important for explaining the cause of this phenomenon.

As can be seen from this comparison, the two comets exhibit extreme similarity. Except in their size and its effect on the /204
clarity of all phenomena, they appear to differ only in that that of 1744 exhibited the above more persistently than the other. Without doubt, however, this stems from the oscillating motion of the latter's emanation, whose great influence on the vitality of the emanation has already become apparent from my observations. Although Heinsius does not determine the directions of the emanation in greater detail, we can still conclude from its persistence that its oscillations were probably of much smaller extent than those of the comet of 1835.

Section 7

I have been able to make an observation which provides a contribution to knowledge about the nature of the nebulosity immediately surrounding the nucleus of the comet. The comet passed very close to a fixed star of tenth magnitude on September 29, and I made use of this opportunity to study whether the

light of the star is deflected by the nebulosity from its linear motion. The type of observation which I made consists of the measurements of a series of distances and positional angles of the eclipsed star, referred to another star close to it which nevertheless remained outside the nebulosity. I had previously compared the position of the comet with the same star by means of 12 similar observations and could thus calculate the distance, at the time of an observation of the eclipsed star, between the comet and both the true position of the star and that which was observed. The difference in the two would have to have disclosed any refraction that occurred. We could expect to achieve a great deal of certainty in this manner, since comparisons between two fixed stars can be made more accurately than comparisons between the comet and a fixed star, particularly with one that was as weak, compared with the comet, as the eclipsed star was.

The reference star (a) is of eighth magnitude and occurs in Nos. 449 and 490 of my zones; its apparent position obtained from this is

99°7'25.6" and 36°48'23.6"

The eclipsed star (b) was compared 10 times with this assumed position, and the following was obtained from this:

MT	AR.	Decl.
12h 39' 41"	98°49'23"32	36°52'34"12
46 40	24,36	34,51
50 19,6	21,36	34,10
53 6,1	23,43	33,60
59 11,1	24,04	25
13h 8' 37"	98°49'23"52	36°52'35"14
16 35	23,72	33,18
20 5	2..86	35,47
22 34	26,17	34,76
25 34	22,07	34,90

/205

At the times of the third, fourth and fifth observations, the star was in the denser portion of the comet's nebulosity; the others, during which it was free of the latter, yield its true location:

98°49'23.94" and 36°52'34.58".

In order to be able to reduce the comet observations and to compare its position both with this true position and with the apparent positions of star b observed at the time of its closest approach to the latter, I have calculated several of the comet's positions from the elements already utilized in Section 3, which I present here:

MT Paris.	AR.	Decl.	log ρ
8h 20' 17"2	98° 39' 33"43	36° 42' 46"61	9,74024
11 20 14,9	47 14,15	36 51 39,09	9,73664
14 20 12,7	55 2,32	37 0 39,73	9,73298

The observations of the comet indicated its apparent position:

11:00:53 MT, 98°44'38.95", 36°46'51.19",

and the error in the elements, taking parallax into consideration:

in AR -70.40", in Decl. + 2.96".

In order to evaluate the quality of this determination, I present a comparison of it with the 12 individual measurements on which they are based, namely the remaining differences in these, both in terms of distance and perpendicular to this:

+ 4"41	— 0"95
— 3,11	+ 1,71
+ 1,93	— 0,57
— 1,99	+ 2,17
+ 0,25	— 1,71
— 1,51	— 0,60
— 1,21	— 0,48
+ 0,04	+ 1,06
+ 2,96	— 0,45
— 1,55	+ 0,01
+ 2,27	— 3,37
— 2,51	+ 3,22

It can be seen from a comparison of the apparent position of the comet and the true position of the star that the former was closest to the latter at a distance of 6.66" at 12:54:15 MT. Its apparent positions for the times of the third, fourth and fifth observations are:

98°49'20"72	36°52'19"01
27,86	27,36
43,52	45,67

and its distances from the lower position of the star: /206

15.78"; 7.78"; 19.19".

The distances between the comet and the positions of the star observed at the same times, on the other hand, are

15.10"; 7.18"; 19.32".

If we were to attribute the differences in the two to refraction in the comet's atmosphere, we would obtain twice the value for this:

-0.68"; -0.60"; +0.13".

Thus it can be seen that no refraction was noticeable at the distance from the center at which the observations were made.

The small differences even have opposite signs to those for refraction in two cases and therefore failed to indicate even the slightest trace of the latter. I make use of this opportunity to note that refraction in the comet's atmosphere could be demonstrated in still another manner. If its maximum is designated R and the apparent distance from the center of the comet at which it occurs is designated ρ, we must see the refracted light of the star at the margin of the comet opposite it when its true distance is $2R - \rho$; at smaller distances, the comet would refract even more light from the star to the eye. At these distances, we must thus -- if refraction occurs -- observe brightness in the comet derived from the star, i.e. see the comet to be brighter than it would appear without the proximity of the star. Prior to the comet's appearance, I hoped to have more frequent opportunities to observe its passages in front of fixed stars in detail, but only the one described occurred. I also believed that there would be an opportunity to note whether the proximity of bright stars increased the light from the comet, but the frequently semiovercast condition of the sky generally destroyed these opportunities; after the development of the emanation, the light from the comet was so strong that later one could no longer expect to observe something of this nature, even if it had come close to very bright stars.

The observation described shows with certainty that the nebulosity, still quite dense at a distance of 7.78" from the center, manifested no refraction. If we wish to conclude from this that the nebulosity exhibits just as little refracting power at even smaller distances than the above, then we can hardly view it as a gaseous fluid, but rather only as consisting of separate particles; at least, we are lacking an analog consisting of a gas which does not affect light.

Section 8 /207

During the above passage of the comet in front of the star, as in the case of several other passages which were not so close, the stars experienced a considerable attenuation of their light. To several observers of earlier comets, stars appeared to have unattenuated brightness in the nebulosity of the comets. Olbers, on the other hand, observed that stars of eighth magnitude appeared considerably weaker when in the bright margin of the tail of the comet of 1811 (Mon. Corr. XXV, p. 15). I likewise have no doubt that I saw stars more weakly in the nebulosity of the comet than outside of it; nevertheless, this cannot convince us as to whether the attenuation of light might not be attributable to the bright background on which the stars appeared. A decisive point in this question appears to me to be an important observation by Arago; I am indebted to Olbers for graciously reporting this information. Mr. Arago studied whether the comet contained polarized light, and found that it really possessed such light. Since this can only occur if the comet reflects light, there can be no further doubt that it attenuates the light of stars as it passes through, for reflection of the light demonstrates that it does not pass through unobstructed.

The observation by Arago just described is an important contribution to information on the comet, since it permits no doubt that it reflects sunlight. Nevertheless, even though the comet receives all of the light which it exhibits from the sun, only a small portion of it is polarized, and it does not follow, from that fact that it contains polarized light, that the quite larger, unpolarized portion of its light consists entirely of sunlight. I would consider this remark to be superfluous if my observations did not appear to indicate that the comet has generated light of its own. They of course have neither the merit of Arago's nor have they been capable of being made independently

of the variable transparency of the air. Yet it still appears to me as if the great increase in brightness which became visible on October 2, in conjunction with the simultaneous origin of the emanation, may be no more overlooked than the decrease in the nucleus' brilliance noted on October 14 in spite of the excellent clearness of the sky.

Several observers have seen earlier comets pass over stars without losing sight of the latter. If a passage has truly been central and if the atmosphere of the comet has exhibited no refraction, this observation provides grounds for the conclusion /208 that the nucleus of the comet was not an opaque body. I do not actually believe that we can be completely convinced that the observed eclipses were central; I am also of the opinion that the assertion of the complete absence of refraction must be established on the basis of observations by which the apparent position of a star is determined at an even closer distance to the comet's center than is the case with my observation reported in the preceding section. Regardless of this, however, I consider it probable that the nucleus of the comet is not actually a solid body, i.e. not a solid body of the same type as the earth, moon and planets. It must actually be able to easily make the transition to a state of volatility, whereas the above-named bodies do not possess this property, or at least just to a low degree: since its surface exhibits no fixed boundary, it appears to be in such a state; the almost incomprehensibly large space which is filled by the tails of many comets, along with the apparent extreme smallness of their masses, likewise indicates that the matter in comets acquires the property of unlimited expansion. But the mass of the comet cannot originally possess this property; at least, it cannot be a material which possess no density when it experiences no pressure, for such a material would obviously become completely dispersed. I see no difficulty in assuming, however, that comets consist of parts which are merely lacking a little in heat or some other repulsing property which they have to possess in order

to become volatile. That volatility appears earliest precisely on that part of the surface facing the sun and that it is increased by closer approach to the sun and prolongation of its action and extends over a greater and greater portion of the surface can be expected on the basis of this view and also agrees with observation. That volatility can, through the heat loss associated with it, be the means by which a portion of the comet's mass can be protected from dispersion has already been noted by Laplace, if I am not mistaken. Finally, I would also like to note that transparent and opaque, refracting and nonrefracting comet nuclei are compatible with this view.

Section 9

Luckily, the uncertainty in which we necessarily find ourselves in discussing the components of comets does not prevent the implementation of studies on the motion of those particles /209
which separate from them. They are subject to the general laws of the motion of points, which I shall accordingly attempt to apply to them.

First, however, I must recall the view which Newton held concerning the manner in which the tails of comets arise, for in this view, the motion of the particles of a hypothetical ether filling space, rather than the particles of the comet, must be studied. Newton compared the rising of the comet's nebulosity with the rising of smoke in the air; he assumed that refractions and reflections of light caused by the atmosphere of the comet heated the surrounding parts of the ether and made them lighter, so that they rise in the higher, more dense ether and carry portions of the atmosphere with them. Since this is a view held by Newton, it must correspond to the properties of comet tails known to him; it is impossible, however, to reconcile them with phenomena which have been observed since.

I recall the comet of 1811, whose tail exhibited the striking phenomenon of a strip which was separate from the nucleus, curved somewhat parabolically and having its tip toward the sun; I am thinking of the comet which Olbers described in his imaginative paper on comet tails. I also recall the comet of 1807, which exhibited a longer, almost straight tail and a shorter, highly curved one. Finally, let me describe the comet of 1824, which exhibited two tails, several degrees long, one of which was toward the sun and the other of which was away from it. I view these comets as unequivocal evidence against the opinion that the portions of the comet's atmosphere are carried away by heated portions of ether and shall therefore try to determine the motion of the former, not the latter.

If we wish to consider not only the action of the sun but also the action of the comet on a freely moving particle, the task to be solved is obviously the three-body problem, which, if we do not limit its generality, is known to result in insurmountable difficulties. If we consider the effect of the comet to be appreciable only within short distances and if we shall be satisfied with studying the motion of a particle after it has left the comet's sphere of influence, the task is not longer difficult. We actually have good reason to consider the masses of the comets to be almost infinitesimal as compared to those of /210
the planets, to say nothing of the sun, and accordingly to assume that their effect is appreciable or comparable to that of the sun only at quite short distances. What we lose by studying the motion of particles outside the comet's sphere of influence is a knowledge of the theory of their motion within it, knowledge of which we probably would make no use in any case, since the original state of motion of a particle and the forces acting at the surface of the comet will probably remain unknown. For the original state of motion of a particle I shall assume that with which it leaves the comet's sphere of influence rather than with which it leaves the comet itself.

I shall determine a point with respect to the comet at which a particle is located at time t after having left the comet's sphere of influence at time t - τ at a given position, at a given velocity and in a given direction. I do not base the calculations on the condition that the mass with which the sun acts on the particle be equal to the mass 1 with which it attracts the planets and the comet itself; rather, this mass shall be designated μ and shall remain undetermined during the course of calculation. For the sake of brevity, I shall consider only particles which move in the plane of the orbit.

I must start with an explanation of the mass of the sun relating to the particle, which is to remain undetermined. The force with which the sun endeavors to move the particle at distance r is assumed to be μ/rr. If μ has the value 1, the effect is equivalent to the sun's normal attraction; if μ < 1, the former is smaller than the latter; if μ is negative, attraction is replaced by repulsion. In the two cases which differ from normal attraction, however, it is not necessary to attribute a different attractive force to the sun itself or a repulsive force to the particle; the possibility remains of explaining both in terms of the normal attractive force of the sun. If we wish to do this, we must assume an ether which gravitates toward the sun without offering resistance to motion. Under this condition, the relative attractive force with which the sun acts upon a particle of density d in an ether of density d' is /211

$$\frac{d-d'}{drr} = \left(1 - \frac{d'}{d}\right)\frac{1}{rr}.$$

In this way we can explain any attractive force, if smaller than the normal one, and also a repulsive force. Yet the latter has a limit, as studies on the motion of a pendulum in the air and Poisson's admirable analysis of it have shown: the motion of the tail particle in the ether becomes the necessary cause of a

motion of the ether itself, and the moving force of the former is used to set a larger mass in motion than that of the tail particle. For this reason, the accelerating force is smaller than the relative attraction of the sun; it is expressed by

$$\frac{d-d'}{d+k\,d'}$$

where k indicates a positive coefficient whose value is a function of the form of particle which is moved but which we do not yet know, either from analysis or from observations, for the case of a very rapid, progressive motion. For a sphere, which experiences very small oscillations, Poisson found k = 1/2; observations have indicated a considerably larger value, close to 1. The limit of the repulsive force which we can explain on the basis of a nonresistant ether and which is obtained if the density of the moved body is assumed to be infinitesimal when compared with the density of the ether is 1/k. We may hope that the project of the Paris Institute for which a prize is being offered will yield information on a value of k which can be applied in cases similar to the one at hand. If the ether is assumed to have not just mass but also resistance, a maximum in the velocity of a particle moving away from the sun must obviously occur if its density is not infinitesimally small.

Section 10

I shall label the coordinates of the comet at time t, referred to the orbit's apsidal line and the axis perpendicular to it passing through the sun, as x and y and its radius vector and its true anomaly as r and v; at time t - τ, these quantities are labelled x_0, y_0, r_0, v_0; x', y', r', v' and x'_0, y'_0, r'_0, v'_0 have the same meanings for the particle. If at time t we drop a perpendicular from the particle to the radius vector of the comet and designate the distance between its foot and the comet as ξ, /212

taken to be positive if away from the sun, and the perpendicular itself as η, taken to be positive in the direction from which the comet comes, then

$$\begin{aligned} r\xi &= xx' + yy' - rr \\ r\eta &= yx' - xy' \end{aligned} \qquad (1)$$

must be expressed in terms of τ and the constants necessary for determining the motion of the particle.

For this task we can obtain a finite solution from the known theory of motion about a central body. But this is of no interest here, since the constants upon which the motion of the particle depends must remain explicit in calculation so that we can evaluate the effect which arbitrary hypothetical values for these constants and the quantity μ have on the position of the particle. If we limit ourselves to small values of τ, we can expand the solution in powers of this quantity, this approach appearing to me to be the most desirable one. I shall carry the expansion through τ^3.

From Taylor's theorem, the coordinates of the comet at time $t - \tau$ are

$$\left.\begin{aligned} x_0 &= x - \frac{dx}{dt}\tau + \frac{d^2x}{dt^2}\cdot\frac{\tau^2}{2} - \text{etc.....} \\ y_0 &= y - \frac{dy}{dt}\tau + \frac{d^2y}{dt^2}\cdot\frac{\tau^2}{2} - \text{etc.....} \end{aligned}\right\} \qquad (2)$$

If we designate the relative coordinates of the particle at the same time as

$$\begin{aligned} x'_0 - x_0 &= a \\ y'_0 - y_0 &= b \end{aligned}$$

we then have

$$\left.\begin{aligned} x'_0 &= a + x - \frac{dx}{dt}\tau + \frac{d^2x}{dt^2}\frac{\tau^2}{2} - \text{etc.}\ldots \\ y'_0 &= b + y - \frac{dy}{dt}\tau + \frac{d^2y}{dt^2}\frac{\tau^2}{2} - \text{etc.}\ldots \end{aligned}\right\} \qquad (3)$$

If we also designate the relative velocity of the particle at the instant it leaves the comet's sphere of influence, i.e. at time $t - \tau$, as

$$\begin{aligned} \frac{dx'_0}{dt} - \frac{dx_0}{dt} &= \alpha \\ \frac{dy'_0}{dt} - \frac{dy_0}{dt} &= \beta \end{aligned}$$

we then have

$$\left.\begin{aligned} \frac{dx'_0}{dt} &= \alpha + \frac{dx}{dt} - \frac{d^2x}{dt^2}\tau + \frac{d^3x}{dt^3}\frac{\tau^2}{2} - \text{etc.}\ldots \\ \frac{dy'_0}{dt} &= \beta + \frac{dy}{dt} - \frac{d^2y}{dt^2}\tau + \frac{d^3y}{dt^3}\frac{\tau^2}{2} - \text{etc.}\ldots \end{aligned}\right\} \qquad (4)$$

By again applying Taylor's theorem, we obtain the coordinates of the particle at time t: /213

$$\left.\begin{aligned} x' &= x'_0 + \frac{dx'_0}{dt}\tau + \frac{d^2x'_0}{dt^2}\frac{\tau^2}{2} + \text{etc.}\ldots \\ y' &= y'_0 + \frac{dy'_0}{dt}\tau + \frac{d^2y'_0}{dt^2}\frac{\tau^2}{2} + \text{etc.}\ldots \end{aligned}\right\} \qquad (5)$$

When these have been found, substituting their expressions into (1) gives the ξ and η being sought.

I shall perform the indicated differentiations only for one of the coordinates, since that which applies to one also applies to the other. The differential equations of motion for the comet

$$0 = \frac{d^2x}{dt^2} + \frac{x}{r^3}$$
$$0 = \frac{d^2y}{dt^2} + \frac{y}{r^3}$$

yield

$$\frac{d^2x}{dt^2} = -\frac{x}{r^3}$$
$$\frac{d^3x}{dt^3} = -\frac{1}{r^3}\frac{dx}{dt} + \frac{3x}{r^4}\frac{dr}{dt},$$

as a result of which (3) and (4) become

$$\left.\begin{aligned} x'_0 &= a + x - \frac{dx}{dt}\tau - \frac{x}{r^3}\frac{\tau^2}{2} + \left(\frac{1}{r^3}\frac{dx}{dt} - \frac{3\tau}{r^4}\frac{dr}{dt}\right)\frac{\tau^3}{6} \\ \frac{dx'_0}{dt} &= \alpha + \frac{dx}{dt} + \frac{x}{r^3}\tau - \left(\frac{1}{r^3}\frac{dx}{dt} - \frac{3x}{r^4}\frac{dr}{dt}\right)\frac{\tau^2}{2} \end{aligned}\right\} \qquad (6)$$

The differential equations of motion for the particle are different from those for the comet in that solar mass is taken <u>/214</u> to be μ rather than 1. We thus obtain

$$\frac{d^2x'}{dt^2} = -\frac{\mu x'}{r'^3}$$
$$\frac{d^3x'}{dt^3} = -\frac{\mu}{r'^3}\frac{dx'}{dt} + \frac{3\mu x'}{r'^4}\frac{dr'}{dt}$$

and if we substitute this, applied to time t - τ, into (5), we obtain

$$x' = x'_0 + \frac{dx'_0}{dt}\tau - \frac{\mu x'_0}{r'^3_0}\frac{\tau^2}{2} - \left(\frac{\mu}{r'^3_0}\frac{dx'_0}{dt} - \frac{3\mu x'_0}{r'^4_0}\frac{dr'_0}{dt}\right)\frac{\tau^3}{6}$$

If for x'_0 and dx'_0/dt here we substitute their expressions (6), we obtain

$$x' = a + x + \alpha\tau + \left(\frac{x}{r^3} - \frac{\mu(x+a)}{r'^3_0}\right)\frac{\tau^2}{2}$$
$$-\left\{\frac{2}{r^3}\frac{dx}{dt} - \frac{6x}{r^4}\frac{dr}{dt} - \frac{2\mu}{r'^3_0}\left(\frac{dx}{dt} + \frac{\alpha}{2}\right) - \frac{3\mu(x+a)}{r'^4_0}\frac{dr'_0}{dt}\right\}\frac{\tau^3}{6}$$

In order to obtain the expansion in complete form, we must still expand r_0' in a series. I shall nevertheless shorten the formula by neglecting quantities on the order of the square of the radius of the comet's sphere of influence, or aa, ab, bb, and in the last term, a and b, too. Thus

$$\frac{1}{r_0'^3} = \frac{1}{r^3} - \frac{3(ax+by)}{r^5} + \frac{3}{r^4}\frac{dr}{dt}\tau$$
$$\frac{1}{r_0'^4} + \frac{dr_0'}{dt} = \frac{1}{r^4}\frac{dr}{dt} + \frac{ax+\beta y}{r^5}.$$

and, finally,

$$x' = x + a + \alpha\tau + \left\{(1-\mu)\frac{x}{r^3} - \frac{\mu a}{r^3} + \frac{3\mu x}{r^5}(ax+by)\right\}\frac{\tau^2}{2} - \left\{\frac{2(1-\mu)}{r^3}\left(\frac{dx}{dt} - \frac{3x}{r}\frac{dr}{dt}\right) - \frac{\mu\alpha}{r^3} - \frac{3\mu x}{r^5}(ax+\beta y)\right\}\frac{\tau^3}{6} \qquad (7)$$

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If we substitute this expression and the similar one for y' into (1) and write the square root of half the latus rectum of the comet's orbit, $\sqrt{p}$, for xdy/dt - ydx/dt, we obtain

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$$\left.\begin{aligned} r\xi &= ax+by+(\alpha x+\beta y)\tau + \left\{\frac{1-\mu}{r} + \frac{2\mu}{3r}(ax+by)\right\}\frac{\tau^2}{2} + \left\{\frac{4(1-\mu)}{3r}\frac{dr}{dt} + \frac{4\mu}{r^3}(ax+\beta y)\right\}\frac{\tau^3}{6} \\ r\eta &= ay-bx+(\alpha y-\beta x)\tau - \frac{\mu}{r^3}(ay-bx)\frac{\tau^2}{2} + \left\{\frac{2(1-\mu)}{r^3}\sqrt{p} + \frac{\mu}{r^3}(\alpha y-\beta x)\right\}\frac{\tau^3}{6}. \end{aligned}\right\} \qquad (8)$$

The four constants which occur in these expressions are best expressed in terms of the radius f of the sphere of influence and the angle F between the radius laid toward the point from which the particle emanates and the radius vector, and in terms of its initial velocity g and the angle G between the direction of its motion and the radius vector. If we measure both angles from the sun and in the direction from which the comet comes, we obtain

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$$a = -f\cos(v_0+F) \qquad \alpha = -g\cos(v_0+G)$$
$$b = -f\sin(v_0+F) \qquad \beta = -g\sin(v_0+G)$$

If we also set

$$x = r\cos v \qquad y = r\sin v$$

and

$$\frac{dr}{dt} = \frac{e\sin v}{\sqrt{p}},$$

formulas (8) become

$$\begin{cases} \xi = -f\cos(v-v_0-F) - g\cos(v-v_0-G)\tau \\ \quad + \left\{\frac{1-\mu}{rr} - \frac{2\mu}{r^3} f\cos(v-v_0-F)\right\}\frac{\tau^2}{2} \\ \quad + \left\{\frac{(1-\mu)4e\sin v}{r^3\sqrt{p}} - \frac{4\mu}{r^3} g\cos(v-v_0-G)\right\}\frac{\tau^3}{6} \\ \eta = -f\sin(v-v_0-F) - g\sin(v-v_0-G)\tau \\ \quad + \frac{\mu}{r^3} f\sin(v-v_0-F)\frac{\tau^2}{2} \\ \quad + \left\{\frac{(1-\mu)}{r^3} 2\sqrt{p} - \frac{\mu}{r^3} g\sin(v-v_0-G)\right\}\frac{\tau^3}{6} \end{cases} \qquad (9)$$

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Since time τ is also contained in v_0, completeness of the expansion requires that we also express v_0 in terms of v and τ, or set

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$$\cos v_0 = \cos v + \sin v\frac{\sqrt{p}}{rr}\tau + \left(\frac{p}{r^4}\cos v - \frac{2e\sin v^2}{r^3}\right)\frac{\tau^2}{2}$$
$$\sin v_0 = \sin v - \cos v\frac{\sqrt{p}}{rr}\tau + \left(\frac{p}{r^4}\sin v + \frac{2e\sin v\cos v}{r^3}\right)\frac{\tau^2}{2}$$

We thereby obtain the completely expanded formulas

$$\begin{cases} \xi = -f\cos F - \left\{g\cos G + f\sin F\frac{\sqrt{p}}{rr}\right\}\tau \\ \quad + \left\{\frac{1-\mu}{rr} - g\sin G\frac{2\sqrt{p}}{rr} - f\cos F\left(\frac{2\mu}{r^3}+\frac{p}{r^4}\right) + f\sin F\frac{2e\sin v}{r^3}\right\}\frac{\tau^2}{2} \\ \quad + \left\{\frac{1-\mu}{r^3}\frac{4e\sin v}{\sqrt{p}} - g\cos G\left(\frac{4\mu}{r^3}+\frac{3p}{r^4}\right) + g\sin G\frac{6e\sin v}{r^3}\right\}\frac{\tau^3}{6} \\ \eta = f\sin F + \left\{g\sin G - f\cos F\frac{\sqrt{p}}{rr}\right\}\tau \\ \quad - \left\{g\cos G\frac{2\sqrt{p}}{rr} + f\sin F\left(\frac{\mu}{r^3}-\frac{p}{r^4}\right) - f\cos F\frac{2e\sin v}{r^3}\right\}\frac{\tau^2}{2} \\ \quad + \left\{\frac{1-\mu}{r^3}2\sqrt{p} + g\sin G\left(\frac{\mu}{r^3}+\frac{3p}{r^4}\right) + g\cos G\frac{6e\sin v}{r^3}\right\}\frac{\tau^3}{6} \end{cases} \qquad (10)$$

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Section 11

I shall now compare this theory with the various phenomena which Halley's Comet and that of 1744 have presented; later I shall also apply it to other comets.

If we first consider only an emanation in the direction of the sun, or set F and G = 0, the expression for η becomes

$$-f\frac{\gamma p}{rr}\tau-\left(\frac{\varepsilon\gamma p}{rr}-f\frac{e\sin v}{r^3}\right)\tau^2 + \left(\frac{1-\mu}{r^3}\gamma p+g\frac{3e\sin v}{r^3}\right)\frac{\tau^3}{3}$$

Since at the time of the observations on Halley's Comet it had still not passed through perihelion, sin v was negative: the first two terms of the expression are therefore negative, and all particles emanated between the time of observation and an earlier one in the direction of the sun precede the comet, if the difference between the two times is not so large that the third term multiplied by its cube becomes larger than the two earlier ones multiplied by the first power and the square. This does not occur, however, as we can easily derive from the expression for ξ, for a value of τ for which a particle is still moving away from the comet in the direction of ξ. The particles approaching /216 the sun must thus all precede the comet.

This explains the curvature of the emanation, which I observed primarily on the right, preceding margin of the latter. But this is not the only cause which produces a curvature in the emanation; the rotating motion of the comet, which the emanation follows, either combines with it or acts against it, depending upon whether it moves from the right side to the left or vice versa. For the emanated particles, which maintain their original angular velocity as they rise, remain behind the direction of the

point from which they originated -- the farther they move from it, the farther back they are. A combining of the two causes occurred on October 8 and 12; they acted against one another on October 14: it is thus not remarkable that the curvature was quite visible on the right side during the first days, whereas it was perhaps equal on both sides on the last of these. This could not be evaluated with greater precision because the emanation was not equally vivid at the two margins of the cone on the 14th; at the weaker left side, it had to mix earlier with the light background on which it lay.

This also explains why more and more emanated matter lay on the right side than on the left and why the boundary of it on the /217 sun side did not appear uniformly convex (cf. end of Section 1). The matter flowing to the right side in greater quantity must actually have caused both phenomena.

The length of the nebulosity on the sun side of the nucleus possesses a relationship to μ and the initial velocity which is worth studying in greater detail. If we abbreviate the expression for ξ (10) with

$$\xi = a + b\tau + c\frac{\tau^2}{2} + d\frac{\tau^3}{6}$$

then those particles which have left the sphere of influence of the comet prior to the time of observation have reached the maximum in ξ, for which

$$0 = \frac{d\xi}{d\tau} = b + c\tau + d\frac{\tau^2}{2}$$

i.e. with which

$$\tau = \frac{R-c}{d} \quad (R = \sqrt{(cc-2bd)}).$$

is associated. If we substitute this value for τ into the expression for ξ, we obtain its maximum, which I shall designate -ε, from the expression

$$\varepsilon + a = \frac{(R-c)^2(2R+c)}{6dd},$$

which we can also write

$$\varepsilon + a = \frac{2}{3}bb\frac{2R+c}{(R+c)^2}$$

since

$$d = \frac{cc-RR}{2b}.$$

If ε is a small quantity, as in the case of Halley's Comet, it is necessary that b also be a small quantity compared to 1 - μ. If we therefore omit anything beyond the square of b and use the expressions for the coefficients, we obtain

$$\varepsilon = f\cos F + \frac{rr}{2(1-\mu)}\left(\frac{g}{r}\cos G + f\sin F\frac{\sqrt{f}}{rr}\right)^2.$$

The maximum value of this expression is the farthest distance of the nebulosity on the sun side of the comet. Since it is found very close for F = 0 and G = 0, we obtain this maximum distance:

$$\varepsilon = f + \frac{rrgg}{2(1-\mu)}$$

This expression assumes that the maximum distance of the nebulosity on the sun side which it indicates is located outside the comet's sphere of influence. In order to be able to get an idea of the basis upon which this can be assumed, I shall seek /218 that distance from the comet at which its attractive force and that of the sun are in equilibrium. I shall assume here that Halley's Comet has the mass which Laplace gave as a limit not reached by the mass of the comet of 1770, namely one five-thousandth of the

mass of the earth. Accordingly, the mass of the comet would be 1/1,785,000,000 solar mass, and the distance being sought would be 1/42,250 of the distance of the comet from the sun; a point located this far from it appeared at a distance of 23" at the time of closest proximity to the earth, whereas the nebulosity extended a distance of at least 4 min. Although this estimate cannot guarantee any certainty that the nebulosity actually extended beyond the comet's sphere of influence, it still does not provide any grounds for assuming the opposite.

One difficulty in comparing the formula with observations is the indefinite boundary of the nebulosity, which does not allow a limit to be given for its distance. Nevertheless, I do not doubt that nebulosity existed even at a distance of 4 min from the nucleus at the time of closest proximity to the earth; this estimate, combined with the distance occurring at that time (log = 9.27038), yields $\varepsilon = 0.0002169$. Since log r = 9.96846 at that time, we obtain the following if we neglect f:

$$\log \frac{\varepsilon\varepsilon}{1-\mu} = 6{,}70028.$$

I shall return to this result later.

Section 12

A phenomenon about whose occurrence my observation of October 22 leaves no doubt but which is indicated even more fully by Heinsius' observations in 1744 is that particles which emanate at acute angles to the radius vector cease to approach the sun as their motion continues, and then begin to move away from it, so that they subsequently move within the tail, which faces away from the sun (§ 6).

If we initially assume that the motion of these particles has proceeded at an acute angle to the radius vector, both initially and as they passed out of the comet's sphere of influence, these observations leave no doubt that $1 - \mu$ had a positive value for both comets, or that μ, if not negative, was still smaller than 1, since the expression for μ contains only a single positive term, /219 namely the first of those multiplied by $\tau\tau/2$, for both of the comets, which had not yet reached their perihelions, i.e. for which ν was negative, for values of F and G between 0 and -90°; for values of F and G between 0 and 90°, it assumes the same character if we arrange it in powers of a quantity τ' differing very little from τ and related to it by the equation

$$\tau = \tau' - \frac{1}{3}\frac{c \sin v}{r\sqrt{p}}\tau'\tau'$$

Since the particles have risen, however, $1 - \mu$ must have had a positive value, such a large positive value that it has outweighed all of the negative terms.

In order to free this conclusion from the assumption that particles which moved at acute angles relative to the radius vector at the beginning of their motion also left the comet's sphere of influence at such angles, we need only remark that their motions can be curved away from the direction toward the sun only if $\mu < 1$. For since they initially move toward the sun and experience a greater attraction than the comet for this reason, /220 even in the most unfavorable case $\mu = 1$, it follows that the angles between their directions and the radius vector can only become more acute, rather than obtuse, even in this case, i.e. even more so in the case $\mu > 1$.

We can thus view the rise of particles emanating at an acute angle with the direction toward the sun as being unequivocal evidence of a positive value for $1 - \mu$. Under the condition that

the nebulosity be outside the comet's sphere of influence on the sun side, the maximum in its distance studied in the above manner provides a second piece of evidence for this, since $\varepsilon - f$ is then necessarily positive.

We can nevertheless move considerably farther, even to a determination of the value of $1 - \mu$, by continuing to pursue the phenomena observed with the comets, namely through a consideration of their tails. I shall therefore apply the theory developed to the form of the tails; first, however, I shall cover the transformation of the expressions for ξ and η which is achieved by the introduction of the above-mentioned quantity designated τ', for in this form the expressions more easily yield the conclusions to be drawn from them. Expressed in terms of τ', we obtain them as follows:

$$\begin{aligned}
\xi = & -f\cos F - \left(g\cos G + f\sin F\frac{\sqrt{p}}{rr}\right)\tau' \\
& + \left\{\frac{1-\mu}{rr} - g\sin G\frac{2\sqrt{p}}{rr} + g\cos G\frac{4e\sin v}{3r\sqrt{p}} - f\cos F\left(\frac{2\mu}{r^3}+\frac{p}{r^4}\right) + f\sin F\frac{10e\sin v}{3r^3}\right\}\frac{\tau'\tau'}{2} \\
& - \left\{g\cos G\left(\frac{4\mu}{r^3}+\frac{3p}{r^4}\right) - g\sin G\frac{14e\sin v}{r^3}\right\}\frac{\tau'^3}{6} \\
\eta = & \; f\sin F + \left(g\sin G - f\cos F\frac{\sqrt{p}}{rr}\right)\tau' \\
& - \left\{g\cos G\frac{2\sqrt{p}}{rr} + g\sin G\frac{4e\sin v}{3r\sqrt{p}} + f\sin F\left(\frac{\mu}{r^3}-\frac{p}{r^4}\right) - f\cos F\frac{10e\sin v}{3r^3}\right\}\frac{\tau'\tau'}{2} \\
& + \left\{(1-\mu)\frac{2\sqrt{p}}{r^4} + g\sin G\left(\frac{\mu}{r^3}+\frac{3p}{r^4}\right) + g\cos G\frac{14e\sin v}{r^3}\right\}\frac{\tau'^3}{6}
\end{aligned}$$

The equation of the curve in which particles are found at time t that had previously left the comet's sphere of influence, always at the same point, at the same velocity and in the same direction, is obtained if we eliminate τ' from these expressions for ξ and η. Since I have already noted that the term multiplied by the square of τ' in the first expression is the same as the one which explains the rising of the particles, and since terms on the order of f and g are multiplied by both the first and the third, /219

that is, the lower and the higher power of τ', it follows that if the rising process takes place not just a long time after passage out of the comet's sphere of influence but even for the values of τ' for which we can assume the expression for ξ to be rapidly converging, I maintain that f and g are small quantities compared to 1 - μ. We thus obtain an approximate value of τ' if we solve the equation /220

$$\xi = a + b\tau' + c\frac{\tau'^2}{2} + d\frac{\tau'^3}{6}$$

in which a, b, c, d are designations for the expressions given above for the coefficients, expanded without regard for their third term, or set

$$\tau' = \frac{R-b}{c},$$

where /221

$$R = \sqrt{\{2c(\xi-a) + bb\}}$$

If we also take the last term into consideration, this expression is further improved slightly and, as an even closer approximation, becomes

$$\tau' = \frac{R-b}{c} - \frac{dRR}{6c^3}$$

If we substitute it into the expression for η, namely /222

$$\eta = a' + b'\tau' + c'\frac{\tau'^2}{2} + d'\frac{\tau'^3}{6}$$

and neglect quantities on the order of the squares and products of f and g, we obtain

$$\eta = a' + \frac{b'}{c} R + \frac{cc' - bd'}{2c^3} R^2 + \frac{d'}{bc^3} R^3$$

and, after substitution of the values for a', b', c', d',

$$\eta = f \sin F + (g \sin G rr - f \cos F \sqrt{p}) \frac{1i}{1-\mu} - \left\{ g \sin G \frac{2r^2 e \sin v}{3\sqrt{p}} + f \sin F \left(\frac{\mu r - 3p}{2} \right) - f \cos F \frac{3}{2} re \sin v \right\} \frac{R^2}{(1-\mu)^2} + \frac{rr\sqrt{p}}{2} \frac{R^3}{(1-\mu)^3} \qquad /221$$

This is the equation of the curve in which, at the time of observation, particles are found which have left the comet's sphere of influence prior to that time with given values of f, F, g, G.

If we apply it to points on this curve which are associated with relatively large values of ξ, i.e. for which the second term of the expression for RR is very small compared to the first, and if, in order to obtain a simpler overview, we neglect quantities on the order of the radius f of the comet's sphere of influence, then the expression for η becomes

$$\eta = g \sin G \frac{r}{\sqrt{(1-\mu)}} \sqrt{(2\xi)} - g \sin G \frac{2re \sin v}{3\sqrt{p}} \cdot 2\xi + \frac{\sqrt{p}}{3r} \frac{(2\xi)^2}{\sqrt{(1-\mu)}}$$

If we divide it by ξ, it yields the tangent of the angle between a point on the curve and the extension of the radius vector, namely

$$\tan \varphi = g \sin G \left\{ \frac{r\sqrt{2}}{\sqrt{(1-\mu)}\sqrt{\xi}} - \frac{4re \sin v}{3\sqrt{p}} \right\} + \frac{2\sqrt{(2p)}}{3r} \frac{\sqrt{\xi}}{\sqrt{(1-\mu)}}.$$

Section 13

I shall now compare this formula with my observation of the tail's direction on October 15, which I gave in Section 1. At that time, the position of the comet was

$$AR = 238°46'20", \quad Decl. = 32°18'26";$$

the position of γ Draconis, from which star the direction passed at a distance of 1/2 degree to the left, was

$$AR = 268"11'37", \quad Decl. - 51"30'52".$$

From this it follows that the distance between the two is 28°46.7', and the star's positional angle at the comet is 39°25', to which 1°2' must be added, however, since the direction of the tail passed at a distance of 30' to the left of the star. At that time, the sun had a positional angle of 229°6 at the comet. /222 Thus the tail formed an angle of 8°39' relative to the direction opposite the sun, toward the side from which the comet approached, the one on which η and ϕ are positive. If we reduce these data to the plane of the orbit, which is done on the basis of the formulas in Section 3, we obtain $\phi = 9°4'$. Since the point on the axis of the tail to which this determination is referred was a distance of 14°23.35'from the comet, we find its position from the comet, using the formulas in Section 3 and assuming $\log \rho = 9.33631$, to be 0.055484 and $\xi = 0.054791$. For the time of observation, moreover, we have

$$\log r = 9.94936$$
$$\nu = -72°8'$$

and, from the known elements of the orbit,

$$\log p = 0.06624; \quad e = 0.98572.$$

With these elements the formula yields

$$\tan g\varphi = g \sin G \left\{ \frac{5,3767}{\sqrt{(1-\mu)}} + 1,0314 \right\} + \frac{0,26762}{\sqrt{(1-\mu)}}$$

If for g sin G we substitute the limits on the values which it has, this formula obviously gives the angles which the margins of the tail make with the extension of the radius vector, i.e. its with dimension, too. If we assume the mean of all values of g sin G to vanish, or if we assume that the comet has not emanated tail matter preferentially toward one side of the radius vector, the last term of the formula applies to the axis of the tail, and we obtain the equation

$$\tan g\, 4^\circ\, 5' = \frac{0.26762}{\sqrt{(1-\mu)}}$$

from which it follows that 1 - μ = 2.812, i.e. μ = -1.812.

Once we have determined this value of 1 - μ, we would be able to find the limits of g sin G if we have observed the tail's width dimension. This did not occur because its margins were /223 too indistinct, and I also attached little importance to their observation; I believe, however, that the tail's width to the extent that it could still be distinguished clearly at the specified distance from the comet, was smaller than 3°, i.e. we do not obtain excessively narrow limits for g sin G as the result of this assumption. If we base our calculations on this, we find them to be ±0.006330.

The information obtained on the value of μ makes an estimate of the value of g possible. The equation at the end of Section 11 based on the extent of the nebulosity on the sun side of the comet, namely

$$\log[gg/(1 - \mu)] = 6.70028,$$

yields log g = 8.5;469 if the value obtained for μ is applied. This g is the initial velocity of the particles, referred to the unit of time, which is the reciprocal of the Gaussian number k. If we use the day as the unit of time, we find the value referred to it to be g = 0.000646, or about 15.5 earth radii.

A comparison of the value of g with the limits on g sin G shows that the comet emanated tail material only close to the direction either toward or away from the sun, or that the farther its direction from the other, the lower the velocity of its emanation became. For if we assume all initial velocities to be the same as the values found for them for emanation at the apex, we find G to be enclosed within the limits ±9°42' or 180° ± 9°42'; if G had wider boundaries or if the comet had emanated matter in all directions, velocity would have had to decrease with increasing sin G, for the product g sin G has the specified narrow limits. An assessment of this is assisted by my observations of October 22 and 25, which show visibly emanating matter at larger angles from the direction of the sun; Heinsius' figures of the same even more distinctly show emanation departing from this direction at almost a right angle, but nevertheless a narrow, greatly extended comet tail. We must conclude from this that emanation velocity has decreased with increasing sine of the angle /224 between its direction and the radius vector.

In order to make the comparison between theory and Halley's Comet complete, I should also note that the almost straight direction of the tail which I observed on October 15 is not compatible with the formula at the end of October 12, granted that it calls for a curvature of the tail in each case. In order to convince ourselves of this, we need only seek the distance at which a point on the axis and at distance 1/2 ξ from the comet was located from the straight line which connects the comet and the observed point in the tail. For this first point,

for the other,

$$\eta' = \frac{\sqrt{p}}{3r}\frac{\xi^3}{\sqrt{(1-\mu)}}$$

$$\eta = \frac{\sqrt{p}}{3r}\frac{(2\xi)^3}{\sqrt{(1-\mu)}};$$

the distance of the first from the straight line constructed toward the other is therefore

$$\eta' - 4\eta = -\frac{\sqrt{p}}{3r}\frac{\sqrt{2-1}}{\sqrt{(1-\mu)}}\xi^3;$$

if we divide it by 1/2 ξ, we obtain the angle which is subtended at the comet by the first point and the straight line constructed toward the second:

$$\frac{-4\sqrt{p}}{3r}\frac{\sqrt{2-1}}{\sqrt{(1-\mu)}}\sqrt{\xi},$$

or

$$-tg\varphi\frac{\sqrt{2-1}}{\sqrt{2}};$$

The product of this angle and half the distance of the point to which the observation refers is close to tne apparent distance between the curved tail and the frequently mentioned straight line. We obtain about 20' in this manner -- a quantity which necessarily had to be adulterated with the indistinctness of the tail, to the point of becoming unrecognizable.

I shall now determine the time which theoretically was taken in the particles' rise to the observed point in the tail. From the formula in Section 12, we have

$$\tau' = \frac{R-b}{c} - \frac{dRR}{6c^3}.$$

and if we neglect second-order quantities, as well as f,

$$\tau' = \frac{r\sqrt{(2\xi)}}{\sqrt{(1-\mu)}}\left\{1 + g\sin G\frac{\sqrt{p}}{1-\mu} - g\cos G\frac{2er\sin v}{3(1-\mu)\sqrt{p}}\right\}$$
$$+ g\cos G\frac{rr}{1-\mu} + \frac{\xi}{(1-\mu)^2}\left\{g\cos G\frac{(4\mu r+3p)}{3} - g\sin G\frac{14}{9}er\sin v\right\}$$

For particles which have moved away from the comet and toward the sun, i.e. for which G = 0, this formula yields $\tau' = 0.18064$, and we thereby obtain $\tau = 0.20177$ or, expressed in days, $\tau = 11.73$. Thus according to this calculation, the particles which were located at the observed point at the time of observation, October 15.23, left the comet between October 3 and 4 -- higher and even more remarkable particles, and thus earlier ones. It is probably only chance that this time almost coincides with the time of visible initiation of the emanation, but we can subsequently be aware of this. /225

Now that I have compared all of the phenomena which Halley's Comet has shown me with theory, I must add more regarding the reliability of the value determined for μ. Although it is not to be expected that such crude observations as are necessary with regard to the direction of a comet's tail allow a precise determination of this value, it can still not be doubted that they provide an approximation to it. An examination of the formula at the end of Section 12 shows immediately that a small or even infinitesimal value of 1 - μ is incompatible with an elongated comet tail which is only slightly curved and is approximately opposite to the sun. The center line of the tail, to which the last term of the formula refers, would form an angle with the radius vector which, although infinitesimal at first, would increase rapidly with ξ, so that not only would its curvature become quite considerable, but its direction would also rapidly approach that line which is perpendicular to the direction of the sun. The first term of the formula would take on a considerable magnitude for an appreciable value of g sin G and thus impart a great width to the tail. We see that small values of 1 - μ are

compatible with more or less round nebulosities surrounding comets, but not with tails such as those exhibited by many comets. Thus we cannot doubt the -- real or apparent (§ 9) -- repulsive force of the sun with respect to the tails of comets.

Section 14

The expression for η in terms of ξ (Section 12) shows immediately that all particles emanating from the comet describe exactly the same curve, once they have exceeded the small values /226 of ξ for which this expression is not applicable, if the product $g \sin G$ has an appreciable value. Those emanating between $G = 0$ and $G = 180°$ then describe one branch of the curve; those emanating between $G = 0$ and $G = -180°$, the other.

The result of the theory appears to be quite remarkable to me, since it completely explains the phenomenon exhibited by the comet of 1811. In the case of the latter, tail particles moved in both branches of an approximately parabolically curved line or in the conoidal surface resulting from the rotation of such a line about the axis. Both are quite compatible with theory, which yields a curve when the emanations occur only in the plane of the orbit and a surface when they proceed in all planes.

Heinsius' observations of the comet of 1744 also show - for an even different reason than that derived in Section 13 from the width of its tail -- that the product $g \sin G$ was practically constant, for the matter emanated at acute angles to the radius vector began to move upward on January 31 in two branches of a curve which lengthened more and more in successive observations without consequently increasing significantly in width. Heinsius' drawings show, as already mentioned in Section 13, that emanations were also present at inclinations relative to the radius vector which approach a right angle: we can thus not ascribe the small width of the shank of the tail to a lack of emanation at greater

angles G, but must explain them in terms of a difference in their velocity in different directions which gives the product g sin G a value which is almost or entirely constant.

Olbers listed several comets which can be assumed -- even on the basis of their imperfect descriptions -- to have been similar to the comet of 1811 in the manner in which the tail was formed. If later observations confirm the relatively frequent occurrence of this mode of formation, the relationship between the velocity and the direction of emanation which the comets of 1744 and 1811 have shown can perhaps be demonstrated to be a general property of comets. But for those comets for which it cannot be demonstrated on the basis of observations, it can likewise not be disputed on their basis, for the linearly rising tail of a comet which does not consist of two separate branches can be explained on the basis of the same law, which exhibits a different outcome /227 only because a constant quantity to which g sin G is equal possesses a smaller value. The striking difference in the forms of tails which comets such as that of 1811 and Halley's Comet have exhibited thus does not call for the assumption of a difference in two basic properties; rather, it is explained just by the difference in magnitude of a constant which is peculiar to each. If this constant is large, the two branches of the tail are very distant from one another; if it is small, they come close to one another and look like an undivided tail.

The condition that g sin G be a constant quantity requires -- expressed in words -- that in spite of their different directions relative to the radius vector, the emanating particles all move away from it at the same velocity. Parallel to the radius vector, they thus possess velocities which have a ratio to the above equal to the ratio of the cotangent of the angle between their direction of motion and the radius vector to the radius. Thus the smaller the angles between the radius vector and the particles' direction of motion, the greater these velocities become; the closer

their direction approaches the direction of the radius vector, the greater the velocity with which the particles move away from the center of the comet in opposite directions. This is purely a result of observations; it is not based on any arbitrary assumption. The observations do not make this so certain that they rule out the possibility of a slight inconstancy in the product $g \sin G$; but they leave no uncertainty about this which permits us to doubt the general correctness of it. We also have an observation which visibly shows a simultaneous emanation of luminous matter at opposite parts of a comet, i.e. demonstrates -- without any reasoning -- motion in opposite directions, toward and away from the sun; this is Heinsius's observation of January 31, 1744.

I believe that what I have presented in this and the preceding sections leaves as little doubt about a second effect of a polar force on the comet, likewise with respect to the sun, as the observed oscillations have made the first doubtful.

Section 15

The comet of 1811 did not preferentially exhibit visible emanation in the direction of the sun; rather, the luminous matter appeared to leave all parts of its surface uniformly; at the same time, the form of its tail indicated that it had a relatively large value of the constant $g \sin G$ associated with it. Such a relatively high velocity of the particles' motion in the direction perpendicular to the radius vector means that the angles between their emanations and the latter generally become larger than in the case of a lower one. I note this because it may perhaps be the reason for the visibility of the emanation from one comet toward the sun, and its invisibility in the case of another; yet it is just this visibility, or the striking difference between the boundaries of the cone emanating toward the sun and the matter

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emanating in other directions which both Halley's Comet and that of 1744 exhibited that makes it much more probable that the two emanations have different causes or may not be explained on the basis of a continuous transition from low to high velocities. Emanation in all directions then appears to be a result of a common effect of the sun on all particles in the comet, whereas visible emanation toward the sun can be ascribed to its preferential effect on the surface facing it. It cannot be doubted that, under this assumption, the two emanations occur at different velocities and can satisfy conditions which are different otherwise.

According to Olbers' assessment, it cannct be doubted that Messier observed two different pairs of branches in the comet of 1769. These are obtained from the formula in Section 12 if we assign two different constant values to the product g sin G, one associated with one portion of the emanating matter, the other to a different portion. It is not possible to pursue this phenomenon beyond its explanation, as a generalization, however, because of the shortcomings in its observation.

The comet of 1807 exhibited a very different phenomenon. From October 22 on, two tails were observed in it, both of which proceeded in the same direction from the core but separated at a distance of 1°: one of them was almost in the direction opposite the sun and practically straight; according to a drawing of October 22 which I find in my Lilienthal journals, I was able to follow this tail for a distance of about 4 1/2° and estimate the positional angle of its direction at about 62°. The other tail could be distinguished only about 3° away; it was highly curved and much wider than the former; the great circle through the comet and the outermost visible point on the axis of this tail had a positional angle of about 83° at the comet. The positional angle of the sun at the same time was 233°48', i.e. the end of one tail appeared to be at an angle of about 8°, the /229

other at about 29°, relative to the opposite direction to it. I have dispensed with pursuing this observation further mathematically, since it is too incomplete; nevertheless, it established the fact, which incidentally was noted by all astronomers of the time. Tails which in the same manner emanate in the same direction from a comet but later separate permit no doubt -- when compared with the formula in Section 12 -- regarding the presence of two different values of the repulsive action of the sun, one of which moves one portion of the emanating matter, the other moving the other portion.

Since, on the basis of the remark at the end of Section 13, the presence of the tails of comets in general allows no doubt to remain regarding the action of a force which is significantly different from the normal attractive force of the sun, and since Halley's Comet, for which I have been able to determine its magnitude, showed it to be a repulsion of almost twice the magnitude of the normal attraction, it can no longer be doubted that those parts of comets which form the tails experience the effect of a repulsive force from the sun. Whether this repulsive force is basically different from the normal solar force or is just a result of the rise of tail particles in a much denser, yet not appreciably resistant ether (Section 9) cannot be decided from this. If we assume the latter explanation, the above-mentioned comet of 1807 shows that tail particles of two different specific densities can be present; if we assume a truly repulsive force, its strength is different for different tail particles.

Other phenomena occur, however, which cannot be explained by assuming a nonresistant ether of appreciably greater density than the comets' tail particles. The oscillating motion of the emanation in Halley's Comet has demonstrated the action of a polar force in the latter; and the same force has manifested its

action in that it drove the emanating particles at accelerated velocity in the direction of the line passing through the sun and the comet, both toward the sun and away from it. These two phenomena cannot be explained by assuming a heavy but nonresistant ether. In addition, the possibility of explaining a force different from the normal attractive force of the sun by means of an ether exists only if it is a small attraction or a repulsion not exceeding 1/k (cf. end of Section 9), but not if it is a greater attraction, an example of which case is provided by the /230 tail of the comet of 1824, directed toward the sun.

To explain, by means of a completely hypothetical cause, those cases which we could possibly explain with the ether hypothesis while we must nevertheless acknowledge a force different from the normal attractive force and adequate to explain the repulsion of particles by the sun would at least violate the first of the three rules on which Newton has based the third book of his Principia. Moreover, this hypothesis would require that while they still belonged to the comet and were subject to the sun's normal attractive force, the tail particles possessed a very high density compared to the ether which, after their separation became very low in terms of the same scale. From all of these reasons which more or less oppose an explanation of the tails of comets in terms of the rising of lighter particles in a heavier ether, this hypothesis appears so improbable, even in those cases in which it cannot be refuted as completely untenable, that we can probably not doubt the general presence, relative to the comet's tail, of a force specifically different from the normal attractive force of the sun.

Section 16

I consider the oscillating motion of the visible emanation from Halley's Comet toward the sun to be a manifestation of the

same force that accelerated the motions of particles leaving the comet parallel to the radius vector in opposite directions. I must still explain how I imagine the connection to be between these phenomena and the repulsive force which the sun has manifested both on particles emanated in its direction and on those located in the tail. The direct observations are not enough for us here. Continued research on all conditions under which polar forces appear, which hopefully will arise from continually developing information on the phenomena dependent upon such forces and from Poisson's powerful analysis, frequently applied to this, must fill the gap which the observations leave open.

My idea of the possibility of a connection among all of the phenomena observed with the comets is nevertheless the following. Any action of a body on another can be broken down into two parts, one of which is the same for all parts of the latter, while the other arises from the differences in the effects on various parts. If the effect is very small at very large distances between the bodies, the first part of it is that which becomes appreciable first as we go from these distances to smaller ones; the other achieves an appreciable magnitude only later. In the case of a comet which approaches the sun at a very great distance, the effect common to all of its parts thus appears first: I assume it to consist of a volatilization of particles polarized repulsively with respect to the sun. The other part of the effect, however, becoming appreciable later, can cause a polarization of the comet itself, and a preferential emanation toward the sun. If observations actually show these phenomena, as was the case with the comet of 1744 and Halley's, it cannot be denied that since it arises from a part of the surface facing the sun, i.e. polarized attractively with respect to the latter, the emanation also possesses the same polarization as the emanating particles which seek to approach the sun. The fact that the emanated particles are still repelled by the sun, as observations show, can perhaps

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be explained by saying that the emanation occurs in an area already filled and thus is continually being filled with matter polarized repulsively with respect to it, as a result of which the opposing polarities compensate for one another, and the farther the emanating particles travel from the nucleus of the comet, the more they lose of their original character and the more they assume the opposite.

All phenomena which I have observed with the comet can be related to one another by this approach. We cannot further justify such views, and I am certainly not qualified to attempt this in the case of mine. But we may pursue them further: from this aspect I think I may remark that the view described makes the tail of the comet of 1824, which was directed toward the sun, appear possible if we assume that this comet presented an exception to the rule in that its polarization began earlier than the surrounding nebulosity polarized repulsively with respect to the sun. In this case, there actually would be no conduction of the polarity of he emanation at an apex to that proceeding at the other, and both emanations could continue unobstructed in their original motions. The observed angle of the direction of the two tails of this comet, or rather its deviation from 180°, is also easily explained by the theory developed above.

I would be able to easily pursue this view even further and thereby relate the details in the observed phenomena; we would also not have to search long if it were necessary to find similarities between these phenomena and those which have been observed in electricity and magnetism. I believe I have come too far already, however, rather than not far enough, and thus permit myself only to point out something which relates to the motion of comets.

Section 17

The emanation from Halley's Comet approximately in the direction of the sun gave it, as I have already explained in describing /232
it (Section 1), the appearance of a burning rocket. It must have also had the same effect on its motion as the burning of a rocket has on the latter's; it must have imparted it with a velocity in the opposite direction from its own. For it is not the comet's center of gravity but only the combined center of gravity of the comet and the emanation which at any instant can describe a conic section in accordance with Kepler's laws; since the emanation renews itself every instant but the emanated matter leaves the comet, the recoil action of the former on the comet's center of gravity must likewise renew itself every instant, or be manifested as an accelerating force. The appearance of vitality of the emanation, or rather the apparent ratio of its mass to the mass of the nucleus, must lead to the opinion that the resultant disturbing force with respect to the comet's elliptical motion can be appreciable. I hope that my series of determinations of the comet's position, which thanks to the power of the large heliometer used were quite accurate, will clarify this after an exact determination of the reference stars and final reduction of the observations. I admit that if no effect of this nature should be found, I would consider it to be more probable that the visible emanation is counteracted by opposite, invisible emanations than that it had no appreciable effect of its own. The painstaking studies of our admiral Argelander on the motion of the comet of 1811 appeared to indicate effects of this nature, which I attempted at that time to explain in a similar manner (p. 77 of Argelander's paper). The much more exact observations of the present comet will be much more powerful in this study than the observations on the comet of 1811.

I would also like to point out that careful observations concerning the tails of comets (which can of course not be applied

to all comets) may become the basis for a judgment concerning the existence of a resistant ether in space. We can easily comprehend that the resistance must manifest itself with incomparably greater strength than it acted on the comets themselves if it acts on particles whose density can only be imperceptable compared with the density of the comet. Careful observations concerning the nucleus, nebulosity and tail of a comet generally promise new insight into astronomical physics. I hope that we shall obtain important contributions to the observations made by me on Halley's Comet from various directions, particularly from the possessers of large telescopes, namely from Councilor of State Struve, Sir James South and Sir John Herschel: those of the latter can achieve an increase in the importance already to be expected for them in that the comet was well visible on the Cape of Good Hope at a time when it was no longer observable under desirable conditions at our northern observatories. If these expected contributions make it necessary to return to this material, I shall not neglect this. I also hope to find spare time to use available descriptions of the tails of several comets to determine the magnitude of the solar force acting upon them.